**CMPT 335 Discrete Structures**

**Final Exam**

There are 10 problems. The total cost of all of them is **130 points**.

Solving any 8 of these problems you will get not less than 95 points and solving any 9 of them you will get over 100 points. Good luck!

**Please, submit your solutions in a single MS Word file named FirstName\_LastName\_Final**.

**Any scanned pictures shall be included in that file and MUST be clearly readable**.

1. (**10 points**) Prove using mathematical induction that

**2**.(**10 points**) Prove using mathematical induction that 

**3**. (**10 points**) Prove the following generalization of the De Morgan’s law using mathematical induction



**4**. (2**0 points**) Verify the correctness of the Selection Sort algorithm (**descending** order) using induction.

Algorithm:

**for** j = 1 to **length** (A)-1

{

**for** i=j+1 to **length (**A)

**if** A[j]<A[i] // if this is true, swap A[i] and A[j]

{

key=A[j]

A[j]=A[i]

A[i]=key

}

}

}

**5**. (**10 points**) Evaluate the following (justify your solution)

1.  in 

[4] + [19] = [23] = [3] in because 23mod5=3

1.  in 

= [76] = [1] in  because 76mod5=1

1.  in 

= [170] = [2] in  because 170mod3=2

1. in 

≡ ≡ [1] ≡ [1] in 

1. in 

≡ ≡ [1] ≡ [1] in 

**6.** (**15 points**) A message was encrypted using a simple encryption method with the encryption function (key) . The following encrypted message was obtained HNRTRNKC Invert the encryption function (key) and decrypt the message. Show your work explicitly (including a decrypted message).

Ciphertext: HNRTRNKC

Decryption using p=3(r-13)mod26

H

r=7

p=3(r-13)mod26

p=3(7-13)mod26

p=-18mod26

p=8

I

N

r=13

p=3(r-13)mod26

p=3(13-13)mod26

p=0mod26

p=0

A

R

r=17

p=3(r-13)mod26

p=3(17-13)mod26

p=12mod26

p=12

M

T

r=19

p=3(r-13)mod26

p=3(19-13)mod26

p=18mod26

p=18

S

K

r=10

p=3(r-13)mod26

p=3(10-13)mod26

p=-9mod26

p=17

R

C

r=2

p=3(r-13)mod26

p=3(2-13)mod26

p=-33mod26

p=19

T

Plaintext: IAMSMART

**7**. (**15 points**) Draw a simple graph  where 

**8**. (**15 points**) Prove isomorphism of the following two simple graphs (verify isomorphism invariants and show 1-to-1 correspondence between vertices explicitly (like A2 = f(A1) )



**9**. (**15 points**) Draw a pseudo graph with the set *V* of vertices, the set *E* of edges, find its adjacency matrix, find degrees of all its vertices and verify the Handshaking Theorem and the Theorem about an adjacency matrix.



**10**. (**10 points**)

a) Draw a tree matching the following parenthetical notation 

b) Create a parenthetical notation for the following tree

C

A

B

D

F

E

G

J

H

K